MATHEMATICS METHODS : UNITS 3 & 4, 2023

Test 3 – (10%)
3.1.9, 3.2.5, 3.3.9 to 3.3.16, 4.1.1 to 4.1.5

QZ

Time Allowed	First Name	Surname	Marks
25 minutes	**MARKING GUIDE**		24 marks

Circle your Teacher's Name:

Mrs Alvaro	Ms Chua	Mrs Fraser-Jones
Mrs Greenaway	Mr Luzuk	Mrs Murray
Ms Narendranathan	Mr Tanday	

Assessment Conditions: *(N.B. Sufficient working out must be shown to gain full marks)*

❖ Calculators: Not Allowed

❖ Formula Sheet: Provided

❖ Notes: Not Allowed

PART A – CALCULATOR FREE

QUESTION 1   (3 marks: 1,2)

Simplify each of the following, writing as a single logarithmic term.

a) $\log a + \log a^{-3} + 2$

$= \log a + \log a^{-3} + \log 100$

$= \log(a \times a^{-3} \times 100)$

$= \log\left(\dfrac{100}{a^2}\right)$

✓ Correct answer

Accept $\log(100a^{-2})$

Teaching point:
In future, simplify further to $2\log\left(\dfrac{10}{a}\right)$

b) $\log_6 125 - \log_6 32 - \log_6 0.4$

$= \log_6\left(\dfrac{125}{32 \times 0.4}\right)$

$= \log_6\left(\dfrac{125 \times 10}{32 \times 4}\right)$

$= \log_6\left(\dfrac{625}{64}\right)$

✓ Shows $\dfrac{125}{32}$
✓ Correct final answer

QUESTION 2 (6 marks: 1,2,3)

Let $x = \log_n 6$ and $y = \log_n 12$.

a) Write $2x - y$ as a single logarithmic term.

$= 2\log_n 6 - \log_n 12$

$= \log_n \left(\dfrac{6^2}{12}\right)$ ✓ Correct answer

$= \log_n 3$

b) Express the following in terms of x and/or y.

i) $\log_n(0.5)$

$= \log_n \left(\dfrac{6}{12}\right)$ ✓ Correct answer

$= \log_n 6 - \log_n 12$

$= x - y$

ii) $\log_n(12n)$

$= \log_n 12 + \log_n n$ ✓ Correct answer

$= y + 1$

c) Determine the exact value of n^{3x}.

Let $k = n^{3x}$,

$$\log_n k = \log_n n^{3x}$$
$$\log_n k = 3x$$
$$\log_n k = 3\,(\log_n 6)$$
$$\log_n k = \log_n 6^3$$
$$k = 216$$
$$\therefore n^{3x} = 216$$

✓ Takes logs
✓ Substitutes for x
✓ Evaluates n^{3x}

Do not penalise if left as 6^3

Alternate method:

$x = \log_n 6$

$n^x = 6$

$n^{3x} = (n^x)^3$

$\quad = 6^3$

$\quad = 216$

✓ Determines n^x
✓ Shows $n^{3x} = (n^x)^3$
✓ Evaluates n^{3x}

Do not penalise if left as 6^3

QUESTION 3 (4 marks: 2,2)
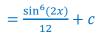
Determine the following,

a) $\frac{d}{dx}\left(3\cos\left(4x + \frac{\pi}{3}\right)\right)$

$= -3\sin\left(4x + \frac{\pi}{3}\right) \times 4$

$= -12\sin\left(4x + \frac{\pi}{3}\right)$

✓ Shows derivative of cos is $-\sin$
✓ Correct answer

b) $\int \cos(2x)\sin^5(2x)\,dx$

$= \frac{\sin^6(2x)}{12} + c$

✓ Anti-differentiates correctly
✓ + C

QUESTION 4 (3 marks)

Determine the equation of the curve with gradient function, $\frac{dy}{dx} = \sin(\pi x)$ at the point $\left(\frac{1}{2}, \pi\right)$.

$y = \int \sin(\pi x)\,dx$

$= \frac{-\cos(\pi x)}{\pi} + c$

✓ Anti-differentiates correctly with +c
✓ Solves for c
✓ States equation of curve

Sub $\left(\frac{1}{2}, \pi\right)$ to solve for c,

$\pi = \frac{-\cos\left(\frac{\pi}{2}\right)}{\pi} + c$

$\pi = 0 + c$

$\therefore c = \pi$

Hence, equation of curve is,

$y = \frac{-\cos(\pi x)}{\pi} + \pi$

QUESTION 5 **(2 marks)**

Decibels are a unit of measure of loudness (of sound) and can be calculated using the equation, $D = 7\log\left(\frac{I}{I_{ref}}\right)$, where I is sound intensity, I_{ref} is the reference sound intensity. What is the decibel level of a sound with intensity one thousand times the reference intensity?

$D = 7\log\left(\frac{1000\,I_{ref}}{I_{ref}}\right)$

$= 7 \times 3$

$= 21\ decibels$

✓ Shows substitution of $I = 1000 I_{ref}$
✓ Correct answer

QUESTION 6 **(6 marks: 2,4)**

Determine the **exact** value of x in each of the following:

a) $5^{x+1} - 5^x = 30$

$5^x(5 - 1) = 30$

$5^x = \frac{30}{4}$

$x = \log_5\left(\frac{15}{2}\right)$

✓ Factorises 5^x
✓ Correct answer

Accept if base 10 used,

$x = \frac{\log\left(\frac{15}{2}\right)}{\log 5}$

b) $\log_2 x = 1 - \log_2(x - 1)$

$\log_2 x = \log_2 2 - \log_2(x - 1)$

$\log_2 x = \log_2\left(\frac{2}{x - 1}\right)$

$\therefore x = \frac{2}{x - 1}$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$\therefore x = 2\ or\ -1$

But $x > 0$, hence $x = 2$

✓ Second line of working needed
✓ Forms & factorises quadratic equation
✓ Obtains both solutions for x
✓ Justifies rejection of -1, and states $x = 2$

END OF PART A

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3.1.9, 3.2.5, 3.3.9 to 3.3.16, 4.1.1 to 4.1.5

QG

Time Allowed	First Name	Surname	Marks
25 minutes	**MARKING GUIDE**		22 marks

Circle your Teacher's Name:

Mrs Alvaro	Ms Chua	Mrs Fraser-Jones
Mrs Greenaway	Mr Luzuk	Mrs Murray
Ms Narendranathan	Mr Tanday	

Assessment Conditions: *(N.B. Sufficient working out must be shown to gain full marks)*

- ❖ Calculators: Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

PART B – CALCULATOR ALLOWED

QUESTION 7 **(5 marks: 1,1,2,1)**

75% of confectionary produced at the Ladbury's chocolate factory are coated in milk chocolate and the rest are coated in dark chocolate. Let X represent the event that a randomly selected chocolate is coated in dark chocolate.

a) Calculate E(X).

$$E(X) = p = 0.25$$

✓ Correct answer

b) Calculate Var(X).

$$Var(X) = p\,(1-p)$$
$$= \frac{1}{4} \times \frac{3}{4}$$
$$= \frac{3}{16}$$

✓ Correct answer

c) If Y = 10X – 2, determine E(Y) and Var(Y).

$$E(Y) = \frac{1}{4} \times 10 - 2 = \frac{1}{2}$$

✓ Correct expected value
✓ Correct variance

Allow follow through

$$Var(Y) = 10^2 \times \frac{3}{16} = \frac{75}{4}$$

d) What is the probability that if three chocolates are selected, at least one will be coated in milk chocolate?

$$1 - P(no\ milk) = 1 - \left(\frac{1}{4}\right)^3 = \frac{63}{64}$$

✓ Correct answer

Allow follow through

QUESTION 8 (6 marks: 1,2,2,1)

A new teaching method to improve arithmetic skills is being investigated by our school. A group of 50 students are randomly chosen to take part in a ten week trial of the new method. There is a 60% chance that any one of these students will show an improvement in arithmetic skills after ten weeks if they do not take part in the trial.

Let X denote the number of students out of the 50 who will show an improvement in arithmetic skills after ten weeks if they do not take part in the trial.

a) State the probability distribution of X.

$$X \sim Bin(50, 0.6)$$ ✓ Correct answer

b) Calculate the mean and standard deviation of X.

$$E(X) = np = 50 \times 0.6 = 30$$

✓ Correct expected value

✓ Correct standard deviation

$$SD(X) = \sqrt{50 \times 0.6 \times 0.4} = 3.464$$

c) What is the probability that at least half of the students will show an improvement in arithmetic skills after ten weeks if they do not take part in the trial?

$$P(X \geq 25) = 0.9427$$

✓ Shows $X \geq 25$

✓ Correct answer

Deduct 1 mark if answer only

d) What is the most likely number of students in a group of 50 to show an improvement after ten weeks if they do not take part in the trial?

Most likely will be expected value, 30.   ✓ Correct answer

QUESTION 9 (5 marks: 2,3)

Bank East has established that 65% of all Visa card accounts are paid in full following the first statement being sent to the customer. A sample of 10 Visa card holders is taken. Let the random variable X be the number of people from the sample of 10 who paid their accounts in full after receiving the first statement.

a) What is the probability that after receiving the first statement, all 10 from the sample pay in full, given that at least 8 paid their account in full?

$$X \sim Bin(10, 0.65)$$
$$P(x = 10 | x \geq 8) = \frac{P(x = 10)}{P(x \geq 8)}$$
$$= \frac{0.0135}{0.2616}$$
$$= 0.05146$$

✓ Shows $\frac{P(x=10)}{P(x \geq 8)}$

✓ Correct answer

b) A new sample group is to be created such that the probability of having at least one Visa card holder having paid their account in full after the first statement is at least 0.95. Determine the minimum sample size.

Y: Number of people in new sample who paid in full
$$Y \sim Bin(n, 0.65)$$

✓ Defines Y~Bin(n, 0.65)

$$P(y \geq 1) > 0.95$$
$$1 - \binom{n}{0} \times 0.65^0 \times 0.35^n > 0.95$$
$$0.35^n < 0.05$$
$$n > 2.85$$

✓ Shows 2nd line of equation

$$\therefore sample\ of\ minimum\ 3\ people$$

✓ Solves n as 2.85 and states 3 people.

Alternate method using CAS > Table
$$Y \sim Bin(n, 0.65)$$
$$P(y \geq 1) > 0.95$$
From CAS table, n = 3 people

Alternate method:
✓ Defines Y
✓ Shows equation (2nd line)
✓ States answer

QUESTION 10

A discrete random variable, X is a Bernoulli distribution as shown.
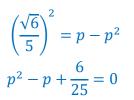
X	0	1
$P(X = x)$	q	p

a) What value of p would maximise the variance, and what is this maximum variance?

$Let\ \sigma^2 = v$

$v = p(1 - p)$
$\quad = p - p^2$

$v' = 1 - 2p$
$v' = 0$
$\therefore p = \dfrac{1}{2}$

$v'' = -2 < 0\ \therefore max$

$v_{max} = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

- ✓ Forms equation for variance
- ✓ Differentiates & solves for p
- ✓ States maximum variance

Accept graphical method as well:
- ✓ Forms equation for variance
- ✓ Finds p at max by showing or describing graph
- ✓ States maximum variance

b) Determine p if $SD(X) = \dfrac{\sqrt{6}}{5}$ and p > q .

$\sigma^2 = p\,(1 - p)$

$\left(\dfrac{\sqrt{6}}{5}\right)^2 = p - p^2$

$p^2 - p + \dfrac{6}{25} = 0$

$\therefore p = 0.4,\quad 0.6$

$since\ p > q,\ \ p = 0.6$

- ✓ Forms appropriate equation
- ✓ Both solutions for p
- ✓ States p > q and hence the answer

END OF PART B